▶ MORENIKEJI NERI, *A metastable Kronecker's lemma with applications to the large deviations in the strong law of large numbers.*
Department of Computer science, University of Bath, Claverton Down, United Kingdom.
E-mail: mn728@bath.ac.uk.

Let $x_1, x_2, ...$ be a sequence of real numbers such that $\sum_{i=1}^{\infty} x_i < \infty$ and let $0 < a_1 \leq a_2 \leq ...$ be such that $a_n \to \infty$. Kronecker's lemma states,

$$\frac{1}{a_n} \sum_{i=1}^{n} a_i x_i \to 0$$

as $n \to \infty$

By applying Godel's Dialectica interpretation, we obtain a finitization of this result as well as the quantitative content of the classical proof of this result in the form of a rate of metastability.

We are then able to use our quantitative results to obtain new rates for the convergence in the strong law of large numbers, for both totally independent (a classic result of Kolmagorov) and pairwise independent sequences of random variables whose distributions are not assumed to be identical, thus, contributing to the study of large deviations in the strong law of large numbers. Furthermore, we are able to better existing rates found in [2].

Lastly, we present a contribution to computability theory, by constructing a sequence of rational numbers that satisfy the premise of Kronecker's lemma but do not converge with a computable rate of convergence (similar to the famous construction of Specker [4]). Thus, we are able to demonstrate the ineffectiveness of Kronecker's lemma. We then show how this ineffectiveness trickles down to the law of large numbers by constructing a sequence of computable random variables, that satisfy the premise of the laws of large numbers we shall study, whose averages do not converge with computable rates.

Our work can be seen as a contribution to the proof mining program, which aims to give a computational interpretation to prima facie non-effective proofs through the application of tools from logic. Our work builds on the new and exciting work on proof mining in probability/measure theory, in particular, [1] and [3].

[1] J. Avigad and P. Gerhardy and H. towsner, *Local stability of ergodic averages*, ***Transactions of the American Mathematical Society***, vol. 362, no. 1, pp. 261–288.

[2] N. LUZIA, *A simple proof of the strong law of large numbers with rates*, ***Bulletin of the Australian Mathematical Society***, vol. 97, no. 3, pp. 513–517.

[3] J. Avigad and E. Dean and J. Rute, *A metastable dominated convergence theorem*, ***Journal of Logic and Analysis***, vol. 4, pp. 3–19.

[4] E. Specker, *Nicht Konstruktiv Beweisbare Sätze der Analysis*, ***Journal of Symbolic Logic***, vol. 14, no. 3, pp. 145–158.